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SUM COMBINATION-2 CORDIAL LABELING OF CERTAIN GRAPHS

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Abstract

Let G be a simple graph and ψ: $V(G) \rightarrow \{1, 2, 3, . . . , |V|\}$ is a bijective mapping. For each edge v_iv_j, assigned the label 1 if $(v_i + v_j)C_2$ is odd and 0 if $(v_i + v_j)C_2$ is even. ψ is called a sum combination -2 cordial labeling if $|e_\psi(0) - e_\psi(1)| \leq 1$, where $e_\psi(0)$ and $e_\psi(1)$ denote the number of edges labeled with 0 and labeled with 1 respectively. A graph which admits a Sum combination-2 cordial labeling is called Sum Combination -2 Cordial graph. In this article, are shown to be Sum Combination-2 Cordial labeling.

Keywords — **Sum combination-2 cordial labeling, Wheel Graph, Graph C_n , Graph $W_n \odot K_2$, Graph $P_n \odot 2K_1$**
AMS Mathematical Subject Classification (2010)- 05C78

I. INTRODUCTION

In this article a new labeling technique introduced by using sum and combination in a different way named it as sum combination -2 cordial labeling. Combination is a technique in which we can make maximum possible arrangements of a system, without repetition. Combination can be used in many real-life situations also. graph labeling is contributed by Rosa in 1967.[6] made the contribution of three different labeling techniques called α labeling, β labeling and γ labeling. Cahit [1] introduced the concept of cordial labeling as a fragile version of graceful and harmonic labeling. Later on there were many more studies happened in the particular area. Gallian [2] made a vast studied on Graph Labeling, which gives an insight to many graph labeling techniques throughout these years. Combination is a technique in which can make maximum possible arrangements of a system, without repetition. Combination technique is applied in graph labeling by Hegde et.al in [3] as there exists a bijection ψ :V(G)→{1,,⋯|v|} such that the induced edge function g:E(G)→N defined as,

$$g(uv) = \begin{cases} \psi(u)C_{\psi(v)}, \; if \; \psi(u) > \psi(v) \\ \psi(v)C_{\psi(u)}, \; if \psi(v) > \psi(u) \end{cases}$$ is injective,

where $\psi(u)C_{\psi(v)}$ is the number of combinations of $\psi(u)$ things taken $\psi(v)$ at a time. Such a labeling ψ is called combination labeling of G. They also proved many graphs holding this labeling technique. Later Murali et.al proved combination cordial labeling and [4] developed this technique into flower and corona graphs. Ponraj [5] contributed parity combination cordial labeling, in which each edge uv, assign the label $\binom{u}{v}$ or $\binom{v}{u}$ according as $u > v$ or $v > u$. ψ is called a parity combination cordial labeling if ψ is a one to one map and $|e_\psi(0) - e_\psi(1)| \leq 1$ where $e_\psi(0)$ and $e_\psi(1)$ denote the number of edges labeled with an even number and odd number respectively. Different types of graph labelling were studied in [7,8,9,10,11,12,13].Motivated from these we introduce a new labeling technique by incorporating both sum and combination called Sum Combination -2 cordial labeling. In this article we obtain Sum Combination -2 Cordial labeling to certain classes of graphs.

Definition 1.1

Let G be a simple graph and $\psi : V(G) \rightarrow \{1, 2, 3, \ldots, n\}$ is a bijective mapping. For each edge $v_i v_j$, assigned the label 1 if $(v_i + v_j)C_2$ is odd and 0 if $(v_i + v_j)C_2$ is even. ψ is called a sum combination -2 cordial labeling if $|e_\psi(0) - e_\psi(1)| \leq 1$. Where $e_\psi(0)$ and $e_\psi(1)$ denote the number of edges labeled with 0 and labeled with 1 respectively. A graph which admits a Sum combination-2 cordial labeling is called sum combination -2 cordial labeling graph.

Example 1.2

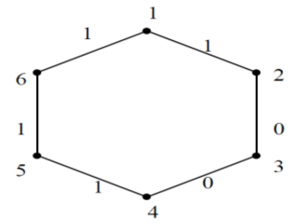


Fig.1.1: Graph C_6

In graph C_6

$(1 + 2)\ C_2 = 3$ $\quad$ $(1 + 6)C_2 = 21$

$(5 + 6)C_2 = 55$ $\quad$ $e_\psi(0) = 4$ and $e_\psi(1) = 2$

$(2 + 3)\ C_2 = 10$ $\quad$ $|e_\psi(0) - e_\psi(1)| > 1.$

$(3 + 4)\ C_2 = 21$

$(4 + 5)\ C_2 = 36$. Therefore, C_6 is not a sum combination-2 cordial graph

Example1.3

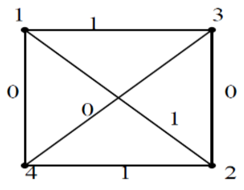


Fig. 1.2: Graph K_4

$(1 + 2)\ C_2 = 3$ $\quad$ $(2 + 3)\ C_2 = 10$

$(3 + 4)\ C_2 = 21$ $\quad$ $(4 + 5)\ C_2 = 36$

$(5 + 6)\ C_2 = 55$ $\quad$ $(1 + 4)\ C_2 = 10$

$e_\varphi(0) = 3$ and $e_\varphi(1) = 3$, $|e_\varphi(0) - e_\varphi(1)| = 0$

Therefore, K_4 is a sum combination-2 cordial graph

Theorem 1.4

The star graph $K_{1,n}$ admits sum combination-2 cordial labeling.

Proof

The star graph $K_{1,n}$ contains (n+1) vertices and n edges. Label $\psi: V(K_{1,n}) \rightarrow \{1,2,3,\ldots,n+1\}$ be the bijection. Let v be the central vertex of degree n. Label the central vertex with 1 and label the remaining pendant vertices with 2,3,4,…n+1. Since $(n+1)C_2$ is even , if $n \equiv 0,1(mod\ 4)$ and odd if $n + 1 \equiv 2,3(mod\ 4)$

Case (1) n is odd

$e_\psi(0) = \frac{n+1}{2}$, $e_\psi(1) = \frac{n-1}{2}$ and $|e_\psi(0) - e_\psi(1)| = 1.$

Case (2) n is even

$e_\psi(0) = \frac{n}{2}$, $e_\psi(1) = \frac{n}{2}$ and $|e_\psi(0) - e_\psi(1)| = 0.$

Hence, $K_{1,n}$ admits sum combination-2 cordial labeling.

Example 1.5

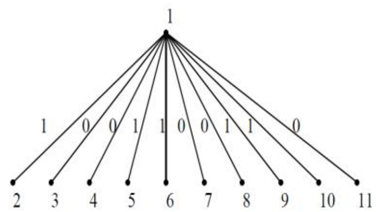


Fig 1.3: The graph $K_{1,10}$

$(1 + 2)\ C_2 = 3$ $\quad$ $(1 + 5)\ C_2 = 15$

$(1 + 3)\ C_2 = 6$ $\quad$ $(1 + 6)\ C_2 = 21$

$(1 + 4)C_2 = 10$ $\quad$ $(1 + 7)C_2 = 28$

$(1 + 8)\ C_2 = 36$ $\quad$ $(1 + 9)\ C_2 = 45$

$(1 + 10)C_2 = 55$ $\quad$ $(1 + 11)C_2 = 66$

$e_\psi(0) = 5$ and $e_\psi(1) = 5$ $\quad$ $|e_\psi(0) - e_\psi(1)| = 0.$

$Hence\ K_{1,10}$ admits sum combination-2 cordial labeling.

Theorem 1.6 .

The path graph P_n admits sum combination-2 cordial labeling

Proof.

The path contains vertex set $\{v1, v2, . . . , vn\}$. Define a bijective function

$\psi : V(P_n) \longrightarrow \{1, 2, . . . , |v|\}$, by $\psi(v_i) = i$ for $1 \le i \le n$. The edges $v_{2i}\ v_{2i+1}$ received the label 0 and edges $v_{2i-1}\ v_{2i}$ received the label 1.

Case (1) n is odd

$e_\psi(1) = \frac{n-1}{2}$ and $e_\psi(1) = \frac{n-1}{2}$

$|e_\psi(0) - e_\psi(1)| = 0.$

Case (2) n is even

$e_\psi(0) = \frac{n}{2} - 1$ and $e_\psi(1) = \frac{n}{2}$

$|e_\psi(0) - e_\psi(1)| = 1.$

Therefore, P_n is a sum combination -2 cordial graph.

2.1 Sum combination-2 cordial labeling of cycle related graphs

Theorem 2.1

The graph cycle C_n, admits sum combination-2 cordial labeling except $n = 6 + 4i$, $i = 0,1,2\ldots,$

Proof

Let G = (V, E) be a simple graph. Define a bijection $\psi: V(C_n) \to \{1,2, \ldots, |v|\}$, by $\psi(v_k) = k$ for $1 \le k \le n.$

Case (1) n is odd, Using Theorem 1.6

For the Path, $e_\psi(0) = \frac{n-1}{2}$ $and\ e_\psi(1) = \frac{n-1}{2}$

For the edge $v_n v_1$, $(1+n)C_2$ is an even number or an odd number.$|e_\psi(0) - e_\psi(1)| \le 1.$

Case (2) n is even, Using Theorem 1.6.

For the Path, $e_\psi(0) = \frac{n}{2}$ $and\ e_\psi(1) = \left(\frac{n}{2} - 1\right) + 1.$

$|e_\psi(0) - e_\psi(1)| = 0.$

Since, (1+n)C_2 is an even number except when $n = 6 + 4i$,$i = 0,1,2, \ldots,$

$If\ n = 6 + 4i, i = 0,1,2 \ldots$, then $v_n v_1$ received the label 1. Hence $|e_\psi(0) - e_\psi(1)| > 1.$

Hence C_n admits sum combination-2 cordial labeling except $n = 6 + 4i$, $i = 0,1,2 \ldots$

Theorem 2.2

The wheel graph W_n admits sumcombination-2 cordial labeling except $= 6 + 4i, i = 0,1,2 \ldots$

Proof

The wheel graph W_n contains (n+1) vertices and 2n edges. Let $\psi: V(W_n) \longrightarrow \{1,2,3 \ldots, |v|\}$ be the bijection defined by $\psi(v_i) = i; 2 \le i \le n + 1$.Label the central vertex v_1 of degree by 1 and the boundary vertices by 2,3,…n+1

Case (1) n is even

Using Theorem 1.4 and Theorem 1.6

$e_\psi(0) = n - 1$ $e_\psi(1) = n$, except when $n = 6 + 4i, i = 0,1,2, \ldots$

$|e_\psi(0) - e_\psi(1)| = 1.$

When $n = 6 + 4i, i = 0,1,2, \ldots$

$e_\psi(0) = n - 1$ $e_\psi(1) = n + 1$

$|e_\psi(0) - e_\psi(1)| = 2 > 1.$

Case (2) n is odd

Using Theorem 1.4 and Theorem 1.6

$e_\psi(0) = n$ and $e_\psi(1) = n - 1$

$|e_\psi(0) - e_\psi(1)| \le 0.$

Therefore, wheel graph W_n admits sum combination-2 cordial labeling except when $n = 6 + 4i$; i = 0,1,2…

Example 2.3

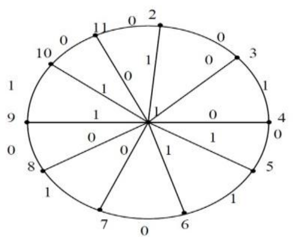


Fig. 2.1: Graph W_{10}

$e_\psi(0) = 12$ $e_\psi(1) = 9$ $|e_\psi(0) - e_\psi(1)| > 1$.
W_{10} is not a sum combination -2 cordial graph.

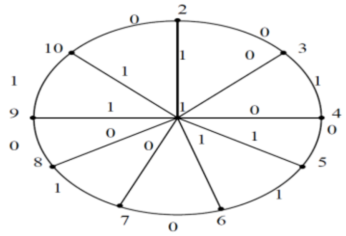


Fig. 2.2: Graph W_9

$e_\psi(0) = 9$ and $e_\psi(1) = 9$, $|e_\psi(0) - e_\psi(1)| = 0$.

W_9 is a Sum combination-2 cordial graph.

Theorem 2.3

The circular ladder graph 〖CL〗_n, n ≥ 2 except when n≡2(mod 4)admits sum combination-2 cordial labeling

Proof.

The circular ladder graph CL_n consists of 2n vertices $\{v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n}\}$ .The vertices on the interior rim are $v_1, v_2, \dots, v_n$,the vertices of the exterior rim are $v_{n+1}, v_{n+2}, \dots, v_{2n}$ and n edges are $v_1 v_{n+1}, v_2 v_{n+2}, \dots, v_n v_{2n}$,in the graph CL_n.

Define a bijection $\psi: V(CL_n) \to \{1,2, \dots, |v|\}$ by $\psi(v_i) = i$ for $1 \le i \le 2n$.

Case (1) $n \equiv 0(mod\ 4)$

$e_\psi(0) = \frac{3n}{2}$ $and\ e_\psi(1) = \frac{3n}{2}$, $|e_\psi(0) - e_\psi(1)| = 0$.

Case (2) $n \equiv 1(mod\ 4)$

$e_\psi(0) = \frac{3n-1}{2}$ $and\ e_\psi(1) = \frac{3n+1}{2}$, $|e_\psi(0) - e_\psi(1)| = 1$.

Case (3) $n \equiv 3(mod\ 4)$

$e_\psi(0) = \frac{3n+1}{2}$ and $e_\psi(1) = \frac{3n+1}{2}$, $|e_\psi(0) - e_\psi(1)| = 0$.

Case (4) $n \equiv 2(mod\ 4)$

$e_\psi(0) = \frac{3n}{2} - 1$ $and\ e_\psi(1) = \frac{3n}{2} + 1$, $|e_\psi(0) - e_\psi(1)| > 1$.

Hence, $CL_n, n > 2$ admits Sum Combination-2 cordial labeling except when $n \equiv 2(mod\ 4)$ admits sum combination-2 cordial labeling.

Theorem 2.4

The comb graph $P_n \odot K_1$ is sum combination-2 cordial labeling.

Proof

The Comb graph $P_n \odot K_1$ contains 2n vertices $(2n-1)$ edges.

Define $\psi: V(P_n \odot K_1) \to \{1,2,3, \dots n, n+1, \dots, |v|\}$ be the bijection defined by

$\psi(v_i) = i;\ 0 \le i \le 2n$

Case (1) n is odd

P_n contains odd number of vertices

Using Theorem 1.6, the graph $P_n \odot K_1$ contains 2n-1 edges label n vertices of P_n by 1,2,…,n and remaining pendant vertices by n+1 n+2,…2n.

The edges $v_{2i}v_{2i+1}$ received the label 0 and the edeges $v_{2i-1}v_{2i}$ received the label 1. Also the edges $v_i v_{n+i}$ received the label 1 for i = 1,3,5… and received the label 0 for

$i = 2,4, \dots, n-1$. Therefore, $e_\psi(0) = $ n and $e_\psi(1) = n-1$.

$|e_\psi(0) - e_\psi(1)| = 1$.

Case (2) n is even

P_n contains even number of vertices

The edges $v_i v_{n+i}$ received the label 0 for $i =$ 1,3,…,n-1 and received the label 1 $for\ i = 2,4,\dots,n$.

Therefore, $e_\psi(0) = \frac{n}{2} - 1 + \frac{n}{2} = n-1$ and $e_\psi(1) = \frac{n}{2} + \frac{n}{2} = n$.

$|e_\psi(0) - e_\psi(1)| = 1$.

$P_n \odot K_1$ admits sum combination-2 cordial labeling.

Theorem 2.5

The Double comb $(P_n \odot 2K_1)$ admits sum combination-2 cordial labeling.

Proof

The Double comb $graph$ $(P_n \odot 2K_1)$ contains $3n-1$ edges and 3n vertices.

Define $\psi: V(P_n \odot 2K_1) \to \{1,2, \dots, |v|\}$ by $\psi(v_i) = i$ for $1 \le i \le n$,$\psi(u_i) = n+i$ for $n+i \le i \le 2n$, and $\psi(w_i) = 2n+i$ for $2n+1 \le i \le 3n$.

The edges $v_{2i}v_{2i+1}$ received the label 0 and the edges $v_{2i-1}v_{2i}$ received the label 1 for the path P_n. Also, the edges $v_i v_{n+i}$ received the label 0 for $i =$

$1,3,5,\dots,n-1$, the edges $v_i v_{n+i}$ received the label 0 for $i=2,4,6,\dots,n$, the edges $v_i v_{2n+i}$ received the label 0 for $i=1,3,5,\dots,n-1$, and the edges $v_i v_{2n+i}$ received 0 label 1 for $i=2,4,6,\dots,n$

Case(1) n is even

Using Theorem 2.4, $e_\psi(0) = \frac{3n}{2} - 1$ and $e_\psi(1) = \frac{3n}{2}$

$|e_\psi(0) - e_\psi(1)| = 1.$

Case (2) n is odd

$e_\psi(0) = \frac{3n}{2} - 1$ and $e_\psi(1) = \frac{3n}{2}$

$|e_\psi(0) - e_\psi(1)| = 1.$

Therefore, $P_n \odot 2K_1$ admits sum combination-2 cordial labeling

Theorem 2.6

The graph $W_n \odot K_2$ admits sum combination-2 cordial labeling.

Proof

The graph $W_n \odot K_2$ contains $3n+1$ vertices and 4n edges. Define a bijection function $\psi: (W_n \odot K_2) \to \{1,2,\dots,|V|\}$ by $\psi(v)=1$ and $\psi(v_i)=i+1, 1\le i\le 3n$, where v_1 is the central vertex of degree n. Label the central vertex 1, the vertices on the wheel by $2,3,\dots,n+1$ and the remaining pendant vertices by $n+2,\dots,3n+1$.

Case(1) n is odd

$e_\psi(0) = 2n$ and $e_\psi(1) = 2n$,

$|e_\psi(0) - e_\psi(1)| = 0.$

Case(2) n is even

$e_\psi(0) = 2n$ and $e_\psi(1) = 2n$,

$|e_\psi(0) - e_\psi(1)| = 0.$

Therefore, $W_n \odot K_2$ is a sum combination-2 cordial graph.

Example

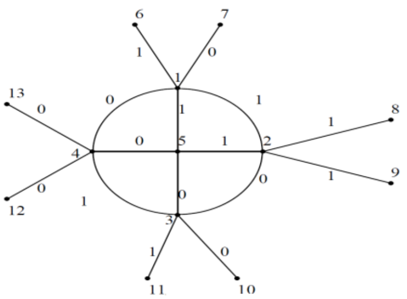


Figure 6.19: Graph $W_4 \odot \overline{K_2}$

$e_\varphi(0) = 8, \ e_\varphi(1) = 8$

$|e_\varphi(0) - e_\varphi(1)| = 0 \le 1.$

3. Sum combination-2 cordial labeling of subdivision of graphs

Theorem 3.1

The subdivision of pendant vertices of a comb graph $Sd(P_n \odot K_1)$ admits sum combination -2 cordial labeling

Proof

The subdivisions of pendant vertices of a comb graph $Sd(P_n \odot K_1)$ contains 3n vertices and $3n-1$ edges . Define a bijection $\psi: V(Sd(P_n \odot K_1) \to \{1,2,\dots,|v|\}$ by $\psi(v_i)=i; 1\le i\le n$. for the path P_n vertices of degree 2, $\psi(v_{i+1}) = n+1, 1\le i\le n$, and pendant vertices by $\psi(v_i)=2n+i; 1\le i\le n$.

Case (1) n is odd

$e_\psi(0) = \frac{3n-1}{2}$ and $e_\psi(1) = \frac{3n-1}{2}$, $|e_\psi(0) - e_\psi(1)| = 0.$

Case (2) n is even

$e_\psi(0) = \frac{3n}{2} - 1$ and $e_\psi(1) = \frac{3n}{2}$, $|e_\psi(0) - e_\psi(1)| = 1.$

Therefore, $Sd(P_n \odot K_1)$, is a sum combination-2 cordial graph

Theorem 3.2

The subdivision of path of a triangular snake graph $Sd(TS_n)$ admits sum combination -2 cordial labeling.

Proof

The subdivision of path of a triangular snake $Sd(TS_n)$ contains $(3n-2)$ vertices and $(4n-4)$ edges. Define a bijection $\psi: V(Sd(TS_n) \to \{1,2,\dots,|v|\}$ by $\psi(v_i)=i$ for $1\le i\le 3n-2$. Label the path $1,2, \dots, (2n-1)$ and remaining vertices by $2n,\dots,(3n-2)$.

Case (1) n is odd

Using Theorem 1.6

$e_\psi(0) = 2n-2$ and $e_\psi(1) = 2n-2$

$|e_\psi(0) - e_\psi(1)| = 0.$

Case (2) n is even

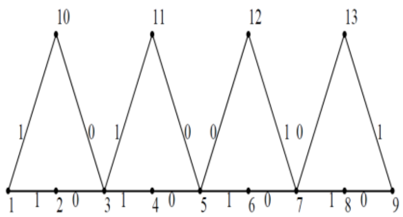
Using Theorem 1.6

$e_\psi(0) = 2n-2$ and $e_\psi(0) = 2n-2$

$|e_{\psi}(0) - e_{\psi}(1)| = 0.$

Therefore, subdivision of path of a Triangular snake graph $Sd(TS_n)$ is a sum combination -2 cordial graph.

Example 3.3



$e_{\psi}(0) = 8, e_{\psi}(1) = 8.$

$$|e_{\psi}(0) - e_{\psi}(1)| = 0.$$

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